UNIT 29 Using Graphs to Solve Equations

CSEC Revision Test

1. Shelly buys 3 litres of oil and 40 litres of gasoline for $30.
   The cost of one litre of oil is $x$ and the cost of one litre of gasoline is $y$.
   (a) Explain why the cost of Shelly's purchases is given by the equation:

   \[ 3x + 40y = 30 \]  

   Craig buys 2 litres of oil and 10 litres of gasoline for his motorbike for $10.
   (b) Write down the equation for Craig's purchases.

   (1 mark)

   (c) Plot these two equations on graph paper like that shown on the next page.

   (3 marks)

   (d) By using your graph, or otherwise, find:

   (i) the cost of one litre of oil,  

   (ii) the cost of one litre of gasoline.

   (1 mark)

2. The dotted lines on the diagram below show part of the graph of \( y = x^3 \).

   (a) On a copy of diagram A, sketch the graph of \( y = x^3 + 1 \).

   (2 marks)

   (b) On a copy of diagram B, sketch the graph of \( y = -x^3 \).

   (2 marks)
Graph paper for Question 1
UNIT 29 Using Graphs to Solve CSEC Revision Test Equations

3. The four sketch graphs below each represent one of the following functions.

\[ y = \frac{2}{x} \quad y = 2x^2 \quad y + \frac{2}{x} = 0 \quad y = -x^2 \]

(a) Copy and complete the table to show which graph represents which function.

<table>
<thead>
<tr>
<th>GRAPH</th>
<th>FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
</tr>
</tbody>
</table>

The line \( y = x \) will intersect three of the four graphs in two places.

(b) State which graph does not intersect the line \( y = x \).

(c) State the coordinates of the points of intersection of \( y = x \) with each of the other graphs.
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4.

(a) The graphs of the lines \( y = x \), \( y = 2x \) and \( y = \frac{1}{2}x \) have been drawn. What is the gradient of the line \( y = x \)? (1 mark)

(b) A rectangle has dimensions \( x \) cm by \( y \) cm. It has an area of 8 cm\(^2\).

(i) Complete the table to show some possible values of \( x \) and \( y \), where \( y = \frac{8}{x} \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.8</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(2 marks)

(ii) Plot these points on a copy of the axes above and draw the graph of \( y = \frac{8}{x} \). (3 marks)
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(iii) A rectangle has an area of 8 cm$^2$. The length $y$ of the rectangle is twice the width $x$. Mark a point on the graph where the dimensions of the rectangle can be found. Label it A. (1 mark)

(iv) A square has an area of 8 cm$^2$. Mark a point on the graph where the length of the side of the square can be found. Label it B. (1 mark)

5. Graph paper must be used for this question.

A rectangular block has a square base of side $x$ cm and a height of $h$ cm. The total surface area of the block is 72 cm$^2$.

(a) Express $h$ in terms of $x$. (2 marks)

(b) Show that the volume, $V$ cm$^3$, of the block is given by $V = 18x - \frac{1}{2}x^3$ (2 marks)

(c) Copy and complete the following table to show corresponding values of $x$ and $V$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>0</td>
<td>40.5</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

(2 marks)

(d) Using a scale of 2 cm to represent 1 unit on the $x$-axis and 2 cm to represent 10 units on the $V$-axis, draw the graph of $V = 18x - \frac{1}{2}x^3$ for values of $x$ from 0 to 6 inclusive. (3 marks)

(e) A block of this type has a volume of 30 cm$^3$. Given that $h > x$, find the dimensions of the block. (2 marks)
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6. (a) The grid shows the line, \( l \), which passes through the points \( A (-1, -2) \) and \( B (2, 4) \).

(i) Determine the gradient of the line, \( l \). (2 marks)

(ii) Write down the equation of the line, \( l \). (1 mark)

(b) (i) Given that \( f(x) = -3 - x^2 \), copy and complete the table below for \(-3 \leq x \leq 2\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-6</td>
<td>2</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(2 marks)

(ii) On a copy of the following grid, draw the graph of \( f(x) = -3 - x^2 \) for \(-3 \leq x \leq 2\). (3 marks)

(iii) Write down the coordinates of the points where the line, \( l \), and the graph \( f(x) = -3 - x^2 \) intersect. (2 marks)
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6. (continued)

(CXC)
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7. The figure provided shows the graph of \( f(x) = x^2 - 9 \) for \(-3 \leq x \leq 3\) and of the graph of \( g(x) = -x - 7 \)

(a) State the elements in the domain of \( f(x) \) for which \( f(x) = -5 \). 

(b) Given that \( f(x) = g(x) \), show that \( x^2 + x - 2 = 0 \).

(c) Solve the equation \( x^2 + x - 2 = 0 \).
8. An athlete runs on a track so that his distance, $d$ metres, from the starting point after $t$ seconds is as shown in the table below.

<table>
<thead>
<tr>
<th>Time (seconds), $t$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (metres), $d$</td>
<td>0</td>
<td>14</td>
<td>40</td>
<td>74</td>
<td>94</td>
<td>100</td>
</tr>
</tbody>
</table>

(a) (i) Using a horizontal scale of 1 cm to represent 1 second and a vertical scale of 1 cm to represent 10 metres, construct a distance-time graph to show the motion of the athlete.

(ii) Draw a smooth curve through all the plotted points.

(b) Use your graph to estimate

(i) the distance travelled by the athlete after 3 seconds

(ii) the average speed of the athlete during the interval $t = 6$ seconds to $t = 8$ seconds.

(iii) the speed of the athlete 6 seconds after leaving the starting point.

(10 marks)

(CXC)

TOTAL MARKS: 68
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ANSWERS

1. (a) Explanation
(b) \(2x + 10y = 10\)
(c) Graph
(d) (i) $2 (ii) 60 cents

2. (a)
(b)

21
01

0
0

3. (a) | Graph | Function |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(y = 2x^2)</td>
</tr>
<tr>
<td>B</td>
<td>(y = -x^2)</td>
</tr>
<tr>
<td>C</td>
<td>(y = \frac{2}{x})</td>
</tr>
</tbody>
</table>
| D | \(y + \frac{2}{x} = 0\)

(b) D

(c) A: (0, 0), (\(\frac{1}{2}, \frac{1}{2}\))
B: (0, 0), (−1, −1)
C: (\(\sqrt{2}, \sqrt{2}\), (−\(\sqrt{2}, −\sqrt{2}\)

4. (a) 1
(b) (i)

<table>
<thead>
<tr>
<th>x</th>
<th>0.8</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>10</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1.6</td>
<td>1</td>
<td>0.8</td>
</tr>
</tbody>
</table>

(ii) B1 B1

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5. (a) \[ h = \frac{36 - x^2}{2x} \left( = \frac{18 - x}{2} \right) \] (b) \[ V = h \cdot x^2 \] M1 A1 B2

(c) | x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>0</td>
<td>17.5</td>
<td>32</td>
<td>40.5</td>
<td>40</td>
<td>27.5</td>
<td>0</td>
</tr>
</tbody>
</table>

B2

(d) Graph B3

(e) 1.85 B2

(11 marks)

6. (a) (i) Gradient \[ = \frac{4 - (-2)}{2 - (-1)} = 2 \] M1 A1

(ii) \[ y = 2x \] B1

(b) (i) \[ f(-2) = -1, \quad f(0) = 3, \quad f(1) = 2 \] (1 for each mistake) B2

(ii) \[ \text{(iii) } (-3, -6) \text{ and } (1, 2) \] B3

(10 marks)
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7. (a) \( x = -2 \) and \( x = 2 \)  
    
(b) \( x^2 - 9 = -x - 7 \Rightarrow x^2 + x - 2 = 0 \)  
    
(c) From graph, \( x = -2 \) and \( x = 1 \)  

8. 

(a) (i) Axis Points \( \text{B1 B2} \)  

(ii) Curve \( \text{B2} \)  

(b) (i) 25 m \( \text{B1} \)  

(ii) Average speed \( \frac{\text{distance travelled}}{\text{time taken}} = \frac{94 - 74}{8 - 6} = 10 \text{ m/s} \) \( \text{M1 A1} \)  

(iii) Speed \( \frac{\text{increase in distance}}{\text{increase in time}} = \frac{95 - 45}{7.5 - 4} = 14.3 \text{ m/s} \) \( \text{M1 A1} \)  

(10 marks)

TOTAL MARKS: 68